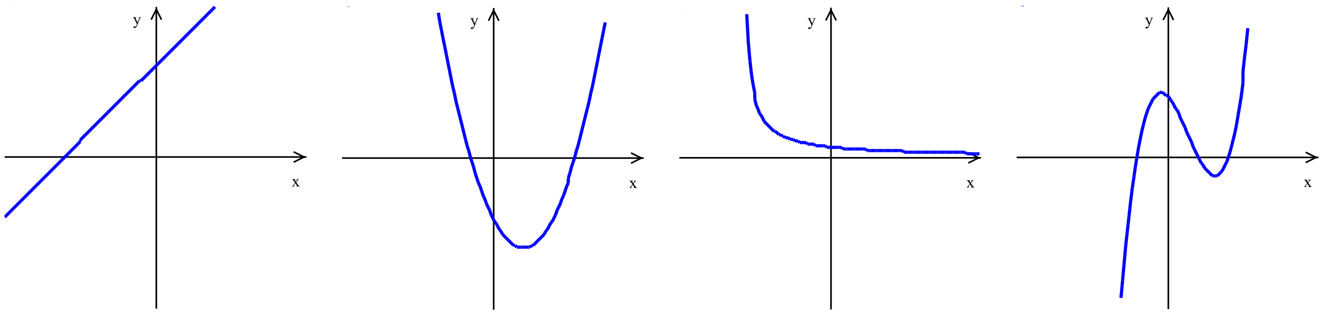
4.1 Inverse Functions

Vertical Line Test: If each vertical line intersects the graph at only one point, then the graph is the graph of a function.



The Horizontal Line Test for Inverse Functions
The function f has an inverse that is a function, f^{-1}, if there is no horizontal line that intersects the graph of the function f at more than one point.

f^{-1} reads "f inverse."

The graphs do not pass the horizontal line test. These are not the graphs of functions with inverse functions.

One-to-One Function: a function in which no two different ordered pairs have the same second component. (The *y*-values are never repeated for other *x*-values.) ***ONLY one-to-one functions have inverse functions.***

Functions that are inverses actually "undo" each other's results.

Ex. A relation in *x* and *y* is given. Determine if the relation defines *y* as a one-to-one function of *x*.

(a) $\{(-14,1),(-2,3),(7,4),(-9,-2)\}$

(b)

X	*Y*
12.5	3.21
5.75	-4.5
2.34	7.25
-12.7	3.21

Definition of the Inverse of a Function

If *f* and *g* are two functions such that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, then the function *g* is the inverse of the function *f* and is denoted by f^{-1}.

Thus, $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$. **The domain of *f* is equal to the range of f^{-1}, and vice versa.**

Ex. Using composition, verify that $f(x)$ and $g(x)$ are inverse functions.

(a) $f(x) = \dfrac{2}{x-5}$ and $g(x) = \dfrac{2}{x} + 5$

(b) $f(x) = 4x + 9$ and $g(x) = \dfrac{x-9}{4}$

If the function *f* is the set of ordered pairs (x, y), then the inverse of *f* is the set of ordered pairs (y, x).

The graph of f^{-1} is a reflection of the graph of *f* about the line $y = x$.

Finding the Inverse of a Function:

1.) Replace $f(x)$ with y.

2.) Interchange x and y.

3.) Solve for y.

4.) Replace y by $f^{-1}(x)$.

Ex. The given functions are all one-to-one.

i) Find the inverse function.

ii) Using composition to verify your equation is correct.

(a) (#42) $g(x)=\dfrac{8-x}{3}$

(b) $f(x)=(x-1)^3$

Ex. (#56) Given $f(x)=\sqrt{x-2}$.

i) Use the graph of f, is f a one-to-one function? _______________

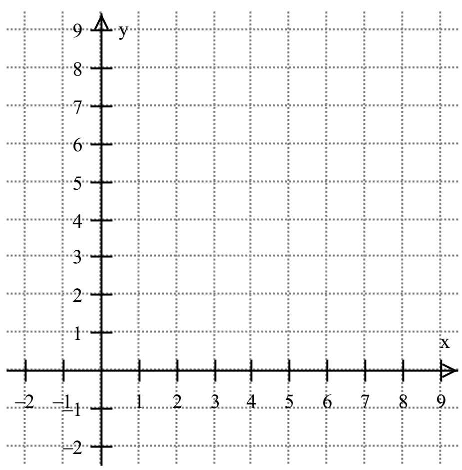
ii) Use interval notation to write the domain and the range of f.

Domain of f: _____________ Range of f: ______________

iii) Find $f^{-1}(x)$.

Note: We need to restrict the domain, so that it is a one-to-one function.

iv) Graph f and f^{-1} in the same rectangular coordinate system.



v) Use interval notation to write the domain and the range of f^{-1}.

Domain of f^{-1}: ____________ Range of f^{-1}: _____________

Ex. Use the graph of f to draw the graph of its inverse function.